

Capital Asset Pricing Model (CAPM)

Chapter 2: Capital Asset Pricing Model

Agenda

- 1 Introduction
- 2 Capital Asset Pricing Model (CAPM)
- 3 Economic interpretations associated with CAPM
- 4 Hedging with CAPM
- 5 Parameter Estimation for CAPM
- 6 Real Applications
- 7 References

Introduction

- In common sense, a **price** is the **amount you need to pay for some goods/services**
- In the context of portfolio selection, a **price** can be regarded as **the return required for bearing a risk**
- **Capital Asset Pricing Model (CAPM)** states that

$$E(R_a) = r + \beta_a (E(R_M) - r)$$

- r is the risk-free rate
- $E(R_a)$ is the **expected return** of an individual **capital asset a**
- $E(R_M)$ is the **expected return** of the **tangency portfolio M (market portfolio)**
 - $E(R_M) - r$ is **market premium**: return needed to compensate the risk of M
- $\beta_a = \frac{E(R_a) - r}{E(R_M) - r}$ is **sensitivity** of expected excess return (asset over market)
 - (will show) $\beta_a = \frac{\text{Cov}(R_a, R_M)}{\text{Var}(R_M)}$ can be regarded as a **measure of risk**
 - high risk $\Leftrightarrow$ large $\beta_a \Rightarrow$ large $E(R_a) \Leftrightarrow$ high return

Capital Asset Pricing Model (CAPM)

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Some investments do have higher expected returns than others. Which ones? Well, by and large they're the ones that will do the worst in bad times.

— *William Forsyth Sharpe* —

AZ QUOTES

William F. Sharpe (1934~): Nobel Economics 1990

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Capital Asset Pricing Model (CAPM)

- **Market Setting:**

- Return of n assets: $\mathbf{R} = (R_1, \dots, R_n)$
- Expected return: $\boldsymbol{\mu} = E(\mathbf{R})$
- Variance of return: $\text{Var}(\mathbf{R}) = \boldsymbol{\Sigma} = (\sigma_{ij}) = (\text{Cov}(R_i, R_j))$

- **Tangency portfolio** M : $\mathbf{w}_M = \frac{\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - r\mathbf{1})}{c_{1\mu} - rc_{11}}$, $\mu_M = \frac{c_{\mu\mu} - rc_{1\mu}}{c_{1\mu} - rc_{11}}$, $\sigma_M = \frac{|\gamma|}{c_{1\mu} - rc_{11}}$

- $\gamma^2 = c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}$, $c_{11} = \mathbf{1}^T \boldsymbol{\Sigma}^{-1} \mathbf{1}$, $c_{1\mu} = \mathbf{1}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$, $c_{\mu\mu} = \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$

Theorem 1 (CAPM)

For any asset a with return R_a , the expected return $\mu_a = E(R_a)$ can be expressed as

$$\mu_a = r + \beta_a (\mu_M - r) ,$$

- r is the risk-free rate
- $\mu_M = E(R_M)$ is the expected return of the tangency portfolio
- $\beta_a = \frac{\text{Cov}(R_a, R_M)}{\text{Var}(R_M)}$

Capital Asset Pricing Model (CAPM)

- **Tangency portfolio** M : $\mathbf{w}_M = \frac{\Sigma^{-1}(\boldsymbol{\mu} - r\mathbf{1})}{c_{1\mu} - rc_{11}}$, $\mu_M = \frac{c_{\mu\mu} - rc_{1\mu}}{c_{1\mu} - rc_{11}}$, $\sigma_M = \frac{|\gamma|}{c_{1\mu} - rc_{11}}$
- $\gamma^2 = c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}$, $c_{11} = \mathbf{1}^T \Sigma^{-1} \mathbf{1}$, $c_{1\mu} = \mathbf{1}^T \Sigma^{-1} \boldsymbol{\mu}$, $c_{\mu\mu} = \boldsymbol{\mu}^T \Sigma^{-1} \boldsymbol{\mu}$

Thm 1 (CAPM): $\mu_a = r + \beta_a (\mu_M - r)$, where $\beta_a = \frac{\text{Cov}(R_a, R_M)}{\text{Var}(R_M)}$

Proof: Method 1: direct computation of covariance.

- If a market is **complete**, then any asset can be replicated by existing assets
 - ⇒ asset a can be expressed as $\mathbf{w}_a = (w_{a1}, w_{a2}, \dots, w_{an})$
 - ⇒ **return** of asset a is $R_a = \mathbf{w}_a^T \mathbf{R}$. ($\mathbf{R}$ is the return vector of assets in the market)
 - ⇒ **expected return** of asset a is $\mu_a = \mathbf{w}_a^T \boldsymbol{\mu}$, where $\boldsymbol{\mu} = E(\mathbf{R})$
- Computation of variance and covariance:

$$\begin{aligned}
 \text{Var}(R_M) &= \sigma_M^2 = \frac{|\gamma|^2}{(c_{1\mu} - rc_{11})^2} = \frac{c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}}{(c_{1\mu} - rc_{11})^2} = \frac{\mu_M - r}{c_{1\mu} - rc_{11}} \\
 \text{Cov}(R_a, R_M) &= \text{Cov}(\mathbf{w}_a^T \mathbf{R}, \mathbf{w}_M^T \mathbf{R}) = \mathbf{w}_a^T \Sigma \mathbf{w}_M \\
 &= \frac{\mathbf{w}_a^T \Sigma^{-1} (\boldsymbol{\mu} - r\mathbf{1})}{c_{1\mu} - rc_{11}} = \frac{\mathbf{w}_a^T (\boldsymbol{\mu} - r\mathbf{1})}{c_{1\mu} - rc_{11}} = \frac{\mu_a - r}{c_{1\mu} - rc_{11}} \quad (\mathbf{w}_a^T \mathbf{1} = 1) \\
 &= \frac{\mu_a - r}{\mu_M - r} \text{Var}(R_M)
 \end{aligned}$$

- The result follows by straightforward algebra

Capital Asset Pricing Model (CAPM)

- **Tangency portfolio** M : $\mathbf{w}_M = \frac{\Sigma^{-1}(\boldsymbol{\mu} - r\mathbf{1})}{c_{1\mu} - rc_{11}}$, $\mu_M = \frac{c_{\mu\mu} - rc_{1\mu}}{c_{1\mu} - rc_{11}}$, $\sigma_M = \frac{|\gamma|}{c_{1\mu} - rc_{11}}$
- $\gamma^2 = c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}$, $c_{11} = \mathbf{1}^T \Sigma^{-1} \mathbf{1}$, $c_{1\mu} = \mathbf{1}^T \Sigma^{-1} \boldsymbol{\mu}$, $c_{\mu\mu} = \boldsymbol{\mu}^T \Sigma^{-1} \boldsymbol{\mu}$

Thm 1 (CAPM): $\mu_a = r + \beta_a (\mu_M - r)$, where $\beta_a = \frac{\text{Cov}(R_a, R_M)}{\text{Var}(R_M)} =: \frac{\sigma_{aM}}{\sigma_M^2}$

Proof: Method 2: tangency of portfolios formed by i and M .

- Consider a portfolio with weight w on asset a & $(1 - w)$ on tangency portfolio M
 - ① $\mu_w = w\mu_a + (1 - w)\mu_M$
 - ② $\sigma_w^2 = w^2 \text{Var}(R_a) + 2w(1 - w)\text{Cov}(R_a, R_M) + (1 - w)^2 \text{Var}(R_M)$
- The curve $\{(\sigma_w, \mu_w); w \in \mathbb{R}\}$ should be tangent to the efficient frontier line
 - At $w = 0$, the curve takes value $(\sigma_0, \mu_0) = (\sigma_M, \mu_M)$ on the frontier line
 - If the curve crosses frontier, then efficiency of the frontier is contradicted.
- Tangency $\Rightarrow$ slopes of curve and frontier agree
 - Slope of frontier is $\frac{\mu_M - r}{\sigma_M}$ (the frontier line passes $(0, r)$ and (σ_M, μ_M))
 - Slope of curve at (σ_M, μ_M) is $\left. \frac{d\mu_w}{d\sigma_w} \right|_{w=0} = \frac{d\mu_w}{dw} / \left. \frac{d\sigma_w}{dw} \right|_{w=0}$, where by ① & ②
 - $\frac{d\mu_w}{dw} = \mu_a - \mu_M$
 - $2\sigma_w \frac{d\sigma_w}{dw} = \frac{d\sigma_w^2}{dw} = 2w\sigma_a^2 + 2(1 - 2w)\sigma_{aM} - 2(1 - w)\sigma_M^2$
 - Simplifying $\frac{\mu_M - r}{\sigma_M} = \left. \frac{d\mu_w}{d\sigma_w} \right|_{w=0} = \frac{d\mu_w}{dw} / \left. \frac{d\sigma_w}{dw} \right|_{w=0} = \frac{\mu_a - \mu_M}{(\sigma_{aM} - \sigma_M^2) / \sigma_M} \Rightarrow \text{Done}$

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1. Capital Market Line and Security Market Line

Efficient frontier (with risk-free asset): $\mu_* = |\gamma|\sigma_* + r$, where

$$|\gamma| = \sqrt{c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}}, \quad c_{11} = \mathbf{1}^T \Sigma^{-1} \mathbf{1}, \quad c_{1\mu} = \mathbf{1}^T \Sigma^{-1} \boldsymbol{\mu}, \quad c_{\mu\mu} = \boldsymbol{\mu}^T \Sigma^{-1} \boldsymbol{\mu}$$

Thm 1 (CAPM): $\mu_a = r + \beta_a (\mu_M - r)$, where $\beta_a = \frac{\text{Cov}(R_a, R_M)}{\text{Var}(R_M)} =: \frac{\sigma_{aM}}{\sigma_M^2}$

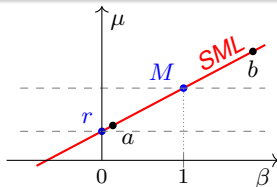
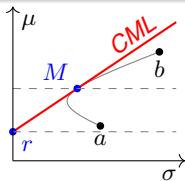
Definition 1 (Capital Market Line & Security Market Line)

- Capital Market Line: μ against σ (same as efficient frontier)

$$\mu = r + |\gamma|\sigma$$

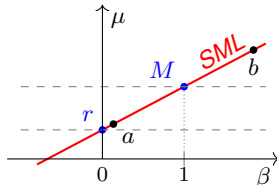
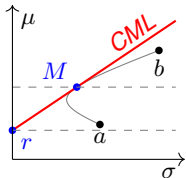
- Security Market Line: μ against β

$$\mu = r + (\mu_M - r)\beta$$



1. Capital Market Line and Security Market Line

- **Capital Market Line** (μ against σ): $\mu = r + |\gamma|\sigma$
- **Security Market Line** (μ against β): $\mu = r + (\mu_M - r)\beta$
 - $|\gamma| = \sqrt{c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}} = (\mu_M - r)/\sigma_M$
 - $c_{11} = \mathbf{1}^T \Sigma^{-1} \mathbf{1}$, $c_{1\mu} = \mathbf{1}^T \Sigma^{-1} \boldsymbol{\mu}$, $c_{\mu\mu} = \boldsymbol{\mu}^T \Sigma^{-1} \boldsymbol{\mu}$



Capital Market Line (CML):

- for each level of risk (in σ), CML gives the **best achievable** expected return μ
- slope of CML = $\frac{\mu_M - r}{\sigma_M}$ is called **market price of risk**
 - represents the **extra return** required for each **additional unit of risk** (in σ)

Security Market Line (SML):

- for each level of risk (in β), SML gives the expected return μ of **any** asset
- slope of SML = $\mu_M - r$ is called **market risk premium**
 - represents the **extra return** required for each **additional unit of risk** (in β)

2. Assumptions in CAPM

- ① Capital markets are perfect
 - No taxes or transaction costs
 - All traders have costless access to all available information
 - Any single participant cannot influence market prices
- ② All investors are mean-variance optimizers.
- ③ All investors have the same estimates of $E(R_i)$, σ_i , & σ_{ij} , for all securities $i, j \in \{1, \dots, n\}$ in the market.
- ④ All investors have the common risk free rate of return r .
- ⑤ Individual assets are infinitely divisible $\Rightarrow$ can hold any units

3. Separation Theorem

One-fund theorem: When risk-free asset is available, the efficient frontier is a linear combination of the tangency portfolio M and the risk-free asset.

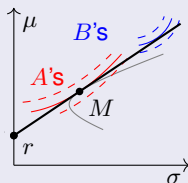
Corollary 2 (Separation Theorem)

The optimal combination of *risky assets* for an investor can be determined without any knowledge of the investor's preferences toward risk and return.

- One's **choice of risky portfolio** and **risk preferences** are **separated**.

Proof.

- From CAPM Assumptions, all investors
 - arrive at the same mean-variance optimizer & tangency portfolio M .
 - invest in the same risk-free asset & same optimal risky portfolio M .
- One's **risk preference** is determined by his **indifference curves**:
 - each investor has his own set of curves. (e.g. two people A & B)
 - indifference curves are convex (increasing return required) due to risk aversion
 - points on the same line represent (σ, μ) -points that are of equal preference
 - higher curves (from the origin) are preferred (higher μ & lower σ)
 - final choice: the highest indifference curve touching efficient frontier $\Rightarrow$ still same M



4. Market Equilibrium

- CAPM model

$$E(R_a) = r + \beta_a(E(R_M) - r)$$

involves **expected values**, representing a state of **equilibrium**

- Properties

- 1 Each security must have a non-zero proportion in the composition of the tangency portfolio.

Proof: Zero proportion $\Rightarrow$ no demand $\Rightarrow$ price drops
 $\Rightarrow$ expected return rises $\Rightarrow$ included in tangency portfolio

- 2 The tangency portfolio is the market portfolio.

- **Market Portfolio**: The weight on security i is

$$w_i^{(m)} = \frac{p_i n_i}{\sum_{i=1}^n p_i n_i} = \frac{\text{market capitalization of security } i}{\text{total market capitalization}},$$

where (p_i, n_i) is (price per share, number of shares) of security i

Proof: Under CAPM, everyone holds the same risky (tangency) portfolio M.
 $\Rightarrow$ The weights on the securities are the same for everyone
But the sum of everyone's holdings is the total market capitalization

4. Market Equilibrium

- The tangency portfolio is the market portfolio.
- **Market Portfolio:** The weight on security i is $[(p_i, n_i)]$ is (price per share, number of shares)

$$w_i^{(m)} = \frac{p_i n_i}{\sum_{i=1}^n p_i n_i} = \frac{\text{market capitalization of security } i}{\text{total market capitalization}},$$

Example 1

Suppose that in a market with n securities and N investors,

- the tangency portfolio is $\mathbf{w}_M = (w_1, \dots, w_n)'$,
- values of tangency portfolio held by the investors are $\mathbf{v} = (v_1, \dots, v_N)'$.

The Market Portfolio is indeed the tangency portfolio:

- Value of security i held by investor j is $w_i v_j$
- Total value of security i in the market is $\sum_{j=1}^N w_i v_j \quad (= p_i n_i)$
- Total market capitalization is $\sum_{i=1}^n \left(\sum_{j=1}^N w_i v_j \right) \quad (= \sum_{i=1}^n p_i n_i)$
- Market portfolio's weight on security i is

$$w_i^{(m)} = \frac{p_i n_i}{\sum_{i=1}^n p_i n_i} = \frac{\sum_{j=1}^N w_i v_j}{\sum_{i=1}^n \left(\sum_{j=1}^N w_i v_j \right)} = \frac{w_i \sum_{j=1}^N v_j}{\left(\sum_{i=1}^n w_i \right) \left(\sum_{j=1}^N v_j \right)} = w_i$$

5. Performance Measures

- The following indexes/ratios evaluate the performance of an asset with
 - expected return μ
 - standard deviation σ
 - beta β

Definition 2 (Sharpe Ratio)

- **Sharpe Ratio** = $\frac{\mu - r}{\sigma}$
- Sharpe ratio for the market portfolio $\frac{\mu_M - r}{\sigma_M}$ is the **Market price of risk**

Definition 3 (Treynor Ratio)

- **Treynor Ratio** = $\frac{\mu - r}{\beta}$
- Treynor ratio for the market portfolio $\frac{\mu_M - r}{\beta_M} = \mu_M - r$ is the **risk premium**
- The higher the ratio, the more valuable the portfolio

6. Systematic and Idiosyncratic risk

- For any asset a and market portfolio M , the CAPM model

$$E(R_a) = r + \beta_a(E(R_M) - r)$$

involves **expected returns**, representing a state of **equilibrium**

- In terms of **expected returns**, we may assume a model

$$R_a = r + \beta_a(R_M - r) + \epsilon_a,$$

where ϵ_a is an error term satisfying

- $E(\epsilon_a) = 0$
- $\text{Cov}(R_M, \epsilon_a) = 0$
- $\text{Var}(\epsilon_a) \geq 0$
- $\sigma_a^2 = \text{Var}(R_a) = \beta_a^2 \sigma_M^2 + \text{Var}(\epsilon_a) = \text{systematic risk} + \text{idiosyncratic risk}$
 - systematic risk**:
 - risk inherent to the entire market
 - also known as “undiversifiable risk”
 - idiosyncratic risk**
 - risk inherent to an individual asset
 - also known as “specific risk” and “unsystematic risk”

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Hedging with CAPM

Theorem 3 (β -hedge)

Consider hedging a portfolio P_t using a *future contract of market portfolio* M_t :

- The future contract trades M_T for $M_0(1+r)$ at time T with no initial cost

1) Borrow \$ M_0 and buy market portfolio; & 2) Hold future contract; give the same payoff

- CAPM model: $E(R_p) - r = \beta_p(E(R_M) - r)$, where $R_p = \frac{P_T - P_0}{P_0}$ and $R_M = \frac{M_T - M_0}{M_0}$

The hedge of the portfolio requires *shorting* $N = \beta_p P_0 / M_0$ future contracts.

Proof.

• Values	$t = 0$	$t = T$
P_t	P_0	$\text{CAPM} \Rightarrow \frac{P_T - P_0}{P_0} - r \approx \beta_p \left(\frac{M_T - M_0}{M_0} - r \right),$ $\Rightarrow P_T \approx P_0(1+r) + \frac{\beta_p P_0}{M_0} [M_T - M_0(1+r)]$
F_t	$F_0 = 0$	$F_T = M_T - M_0(1+r)$

- To reduce randomness (purpose of hedging) $\Rightarrow$ hold $-\frac{\beta_p P_0}{M_0}$ future (short position)

• Hedged Portfolio	$t = 0$	$t = T$
$P_t - \frac{\beta_p P_0}{M_0} F_t$	P_0	$\approx P_0(1+r)$

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Parameter Estimation for CAPM

$$\text{CAPM: } E(R_a) = r + \beta_a(E(R_M) - r) \implies R_a - r = \beta_a(R_M - r) + \epsilon_a$$

Estimation of β_a by regression:

① Data: for $t = 1, \dots, T$

- $Y_t = R_{a,t} - r_t$: excess return of asset a at time t
- $X_t = R_{M,t} - r_t$: excess return of a **proxy** of M at time t
 - Obtaining the tangency portfolio M requires the computation

$$\mathbf{w}_M = \frac{\Sigma^{-1}(\boldsymbol{\mu} - r\mathbf{1})}{(c_{1\mu} - rc_{11})},$$

which is infeasible as $\Sigma \in \mathbb{R}^{n \times n}$ is difficult to be estimated accurately

- In practice a market index (e.g. HSI, SP500) serves as a proxy of M

- r_t : risk free rate at time t

② Fit a regression in $\mathbb{R}$ by $\text{lm}(Y \sim X)$:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \quad \text{where } \mathbf{Y} = \begin{pmatrix} Y_1 \\ \vdots \\ Y_n \end{pmatrix}, \mathbf{X} = \begin{pmatrix} 1 & X_1 \\ \vdots & \vdots \\ 1 & X_n \end{pmatrix}, \boldsymbol{\beta} = \begin{pmatrix} \alpha_a \\ \beta_a \end{pmatrix}.$$

- $\hat{\alpha}_a$ is **Jensen index**, measuring **excess return over CAPM forecast**.

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CAPM: R Example

CAPM $\Rightarrow R_a - r = \beta_a(R_M - r) + \epsilon_a \Rightarrow$ Regress $Y_t = R_{a,t} - r_t$ against $X_t = R_{M,t} - r_t$

Notes on variable names: $z[,1]$ = Market index level, $z[,2]$ = Share Price

Example 2 (A) Download available monthly security prices

```
rm(list=ls(all=T))
library("tseries")
library("quantmod") # bug fix for tseries

d0="2000-02-01"; d1="2022-07-01"; Q="AdjClose" # Parameters
n <- 22 * 12 + 6 # Sample size from 22.5 years
Z <- matrix(0, n, 2) # Matrix to hold data
Z[,1] <- get.hist.quote("^HSI", d0, d1, Q, compression="m")
Z[,2] <- get.hist.quote("1038.HK", d0, d1, Q, compression="m")
Z # Check data

#           [,1]      [,2]
# [1,] 17169.44  3.977433
# [2,] 17406.54  5.340591
# [3,] 15519.30  4.593655
```

CAPM: R Example

CAPM $\Rightarrow R_a - r = \beta_a(R_M - r) + \epsilon_a \Rightarrow$ Regress $Y_t = R_{a,t} - r_t$ against $X_t = R_{M,t} - r_t$

Notes on variable names: rf = risk free rate, R = $[R_M \ R_a]_{n \times 2}$

Example 2 (B) Regression

```
rf <- 0.02/100/12 # Assumed risk-free rate per month
R <- diff(as.matrix(Z))/as.matrix(Z[-n,]) # Compute returns
Y<- R[,2]-rf # Define response
X<- R[,1]-rf # Define predictor
model <- lm(Y~X) # Regression
summary(model) # Check outputs:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.010012	0.003409	2.937	0.0036 **
X	0.313223	0.058658	5.340	1.99e-07 ***

```
mu.M<- mean(R[,1]) #  $\hat{\mu}_M = 0.00231$ 
```

```
mu.a<- mean(R[,2]) #  $\hat{\mu}_a = 0.0107$ 
```

```
s.a<- sd(R[,2]) #  $\hat{\sigma}_a = 0.0587$ 
```

● Fitted CAPM model: $E(R_a) - r = 0.01 + 0.313(E(R_M) - r)$; ($r = 0.02/100/12$)

● Jensen index: $\hat{\alpha}_a = 0.01$

● CAPM β : $\hat{\beta}_a = 0.313$

● Estimated Sharpe Ratio $\left(\frac{\mu-r}{\sigma}\right) = \frac{\hat{\mu}_a-r}{\hat{\sigma}_a} = 0.18289$

● Estimated Treynor Ratio $\left(\frac{\mu-r}{\beta}\right) = \frac{\hat{\mu}_a-r}{\hat{\beta}_a} = 0.034256$

● β -hedge: Short $\hat{\beta}_a P_0 / M_0 = \text{model\$ coef}[2] * Z[1,2] / Z[1,1] = 7.256e-05$ futures

Summary

- **CAPM:** $\mu_a = r + \beta_a (\mu_M - r)$, where $\beta_a = \frac{\text{Cov}(R_a, R_M)}{\text{Var}(R_M)} =: \frac{\sigma_{aM}}{\sigma_M^2}$
 - In terms of returns: $R_a - r = \alpha_a + \beta_a (R_M - r) + \epsilon_a$
 - Decompose $\text{Var}(R_a)$ as **systematic** [$\text{Var}(\beta_a (R_M - r))$] & **idiosyncratic** [$\text{Var}(\epsilon_a)$] risks.
 - Estimate α_a and β_a by regressing $\{R_{a,t} - r_t\}$ against $\{R_{M,t} - r_t\}$
- Graphs in portfolio theory
 - **Capital Market Line (CML):** $\mu = r + |\gamma|\sigma$ (μ against σ); for **optimal** portfolios
 - **Security Market Line (SML):** $\mu = \beta(\mu_M - r) + r$ (μ against β); for **all** portfolios
- Pricing risk:
 - **Jensen index** $= \hat{\alpha}_a$
 - **Sharpe ratio** of security $a = \frac{\mu_a - r}{\sigma_a}$
 - **Treynor ratio** of security $a = \frac{\mu_a - r}{\beta_a}$
 - **Market price of risk** $|\gamma| = (\mu_M - r)/\sigma_M$
 - **Risk premium** $= \mu_M - r$
- **Separation Theorem:** Choice of risky portfolio and risk preference are separated.
- The tangency portfolio is the market portfolio
- **β -hedge:** Hedge a portfolio P by shorting $N = \beta_0 P_0 / M_0$ future contracts of M .

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References

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