

RSMC4003 2023F Midterm Solution

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1 Part I: Applications

1. a) tseries and quantmod. The following is also accepted.

```
1 library("tseries")
2 library("quantmod")
3 PA<-get.hist.quote(instrument = "0609.HK",
4                     start="1997-07-02", end="2022-12-31",
5                     quote = "AdjClose",compression = "d")
6
```

```
b) 1 library("tseries")
   2 library("quantmod")
   3 PA<-get.hist.quote(instrument = "0609.HK",
   4                     start="1997-07-02", end="2022-12-31",
   5                     quote = "AdjClose",compression = "d")
   6
```

```
c) 1 LA=log(PA[-1]/PA[-length(PA)])
   2
```

```
d) 1 length(LA)
   2
```

```
e) 1 mean(LA)
   2
```

2. a) Denote the desired weighting as $\mathbf{w}_p = (w_1, w_2, w_3)^T$ and $\mathbf{1}$ as a vector of 1 with its dimension implicitly known. Using Theorem 1 in Chapter 1, we have immediately

$$\mathbf{w}_p = \Delta^{-1}(c_{11}\hat{\Sigma}^{-1}\hat{\boldsymbol{\mu}} - c_{1\mu}\hat{\Sigma}^{-1}\mathbf{1})0.07 + \Delta^{-1}(c_{\mu\mu}\hat{\Sigma}^{-1}\mathbf{1} - c_{1\mu}\hat{\Sigma}^{-1}\hat{\boldsymbol{\mu}}), \quad (1)$$

where

$$\begin{aligned} c_{11} &= \mathbf{1}^T \hat{\Sigma}^{-1} \mathbf{1} = 0.7826087 \\ c_{1\mu} &= \mathbf{1}^T \hat{\Sigma}^{-1} \hat{\boldsymbol{\mu}} = 0.04782609 \\ c_{\mu\mu} &= \hat{\boldsymbol{\mu}}^T \hat{\Sigma}^{-1} \hat{\boldsymbol{\mu}} = 0.003678261 \\ \Delta &= c_{11}c_{\mu\mu} - c_{1\mu}^2 = 0.0005913043. \end{aligned}$$

Therefore, $\mathbf{w}_p = (-0.0392, 0.598, 0.441)$. The volatility of the portfolio is $\sigma_p^2 = \Delta^{-1}(0.07^2 c_{11} - 2c_{1\mu} \times 0.07 + c_{\mu\mu}) = 1.382353$. $\sigma = 1.175735$.

Students will be given 6 marks at most if all the numerical answers are wrong, 1 mark for each formula.

- b) From Theorem 2, the expected return and variance of overall minimum variance portfolio are given by

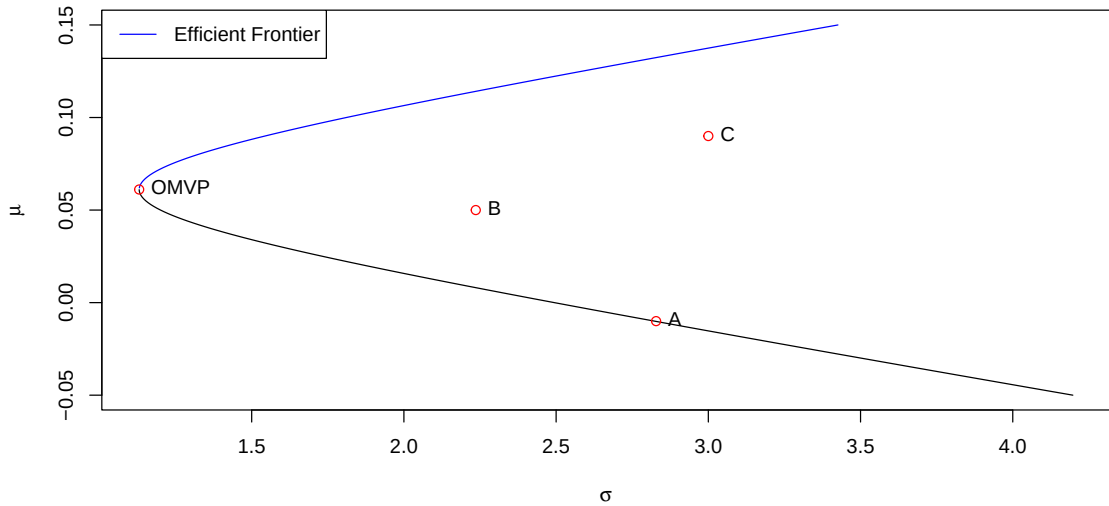
$$\begin{aligned} \mu_{\text{OMVP}} &= \frac{c_{1\mu}}{c_{11}} = 0.06111111 \\ \sigma_{\text{OMVP}}^2 &= \frac{1}{c_{11}} = 1.277778 \\ \sigma_{\text{OMVP}} &= 1.130388. \end{aligned}$$

The Efficient Frontier is the upper part of the minimum-variance set, which is given by

$$\sigma_p^2(\mu_p) = \Delta^{-1} \left\{ c_{11} \left(\mu_p - \frac{c_{1\mu}}{c_{11}} \right)^2 + c_{\mu\mu} - \frac{c_{1\mu}^2}{c_{11}} \right\}, \quad \mu_p > \mu_{\text{OMVP}} = 0.0611.$$

Alternatively

$$\sigma_p^2(\mu_p) = 1323.529\mu_p^2 - 161.7647\mu_p + 6.220588, \quad \mu_p > \mu_{\text{OMVP}} = 0.0611.$$



c) The OMVP is given by

$$\mathbf{w}_{\text{OMVP}} = \frac{\hat{\Sigma}^{-1} \mathbf{1}}{c_{11}} = (0.0741, 0.537, 0.389)^T.$$

```
d) 1 library("quadprog")
    2 D=matrix(c(8,2,-1,
    3           2,5,-4,
    4           -1,-4,9), byrow=1, ncol=3)
    5 )
    6 mu=c(-0.01,0.05,0.09)
    7 d=rep(0,3)
    8 b0=c(0.07,1,d)
    9 A=cbind(mu,one_vector,diag(3))
   10 meq=2
   11 sol=solve.QP(Dmat=D, dvec=d, Amat=A, bvec=b0, meq=meq)
   12 sol$solution
   13
```

-1 mark atmost for any undefined object.

e) According to Theorem 5 in Chapter 1, the tangency portfolio and its mean return and variance are given by

$$\begin{aligned} \mathbf{w}_M &= \frac{\Sigma^{-1}(\hat{\boldsymbol{\mu}} - r\mathbf{1})}{c_{1\mu} - rc_{11}} = (-0.509, 0.851, 0.658)^T \\ \mu_M &= \frac{c_{\mu\mu} - rc_{1\mu}}{c_{1\mu} - rc_{11}} = 0.107 \\ \sigma_M &= \frac{|\gamma|}{c_{1\mu} - rc_{11}} = 2.01 \\ \gamma^2 &= c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11} = 0.00110 \\ \gamma &= c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11} = 0.03316625 \end{aligned}$$

f) Denote the weightings of the new portfolio as (a_D, a_M) such that $a_D + a_M = 1$. Note that the mean return and variance of Asset D are 0.055 and 2.0625 (volatility 1.436141) respectively. Denote the

new portfolio as A.

$$\begin{aligned}\mu_A &= a_D \mu_D + (1 - a_D) \mu_M = \mu_M + a_D (\mu_D - \mu_M) = 0.107 - 0.0518 a_D \\ \sigma_A^2 &= \text{Var}(a_D R_D + (1 - a_D) R_M) \\ &= a_D^2 \sigma_D^2 + (1 - a_D)^2 \sigma_M^2 + 2 a_D (1 - a_D) \text{Cov}(R_D, R_M)\end{aligned}$$

Since both D and M are linear combinations of assets A, B and C,

$$\text{Cov}(R_D, R_M) = \sigma_{D,M} = \mathbf{w}_D^T \Sigma \mathbf{w}_M^T = 0.9078947.$$

Therefore,

$$\sigma_A^2 = \text{Var}(a_D R_D + (1 - a_D) R_M)$$

Let

$$\begin{aligned}\mu_A &= E_1 a_D + E_0 \\ \sigma_A^2 &= V_2 a_D^2 + V_1 a_D + V_0,\end{aligned}$$

where

$$\begin{aligned}E_1 &= -0.0518 \\ E_0 &= 0.107 \\ V_2 &= \sigma_D^2 + \sigma_M^2 - 2\sigma_{D,M} = 4.292417 \\ V_1 &= -2\sigma_M^2 + 2\sigma_{D,M} = -6.275623 \\ V_0 &= \sigma_M^2 = 4.045706.\end{aligned}$$

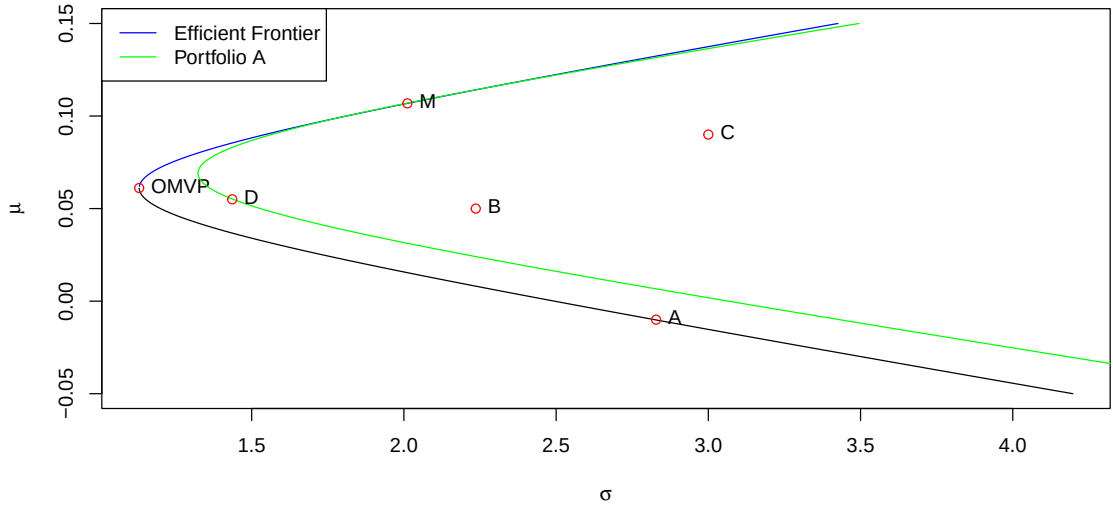
From $\mu_A = E_1 a_D + E_0$, we have $a_D = \frac{\mu_A - E_0}{E_1}$. Substituting it into $\sigma_A^2 = V_2 a_D^2 + V_1 a_D + V_0$ yields

$$\begin{aligned}\sigma_A^2 &= V_2 \left(\frac{\mu_A - E_0}{E_1} \right)^2 + V_1 \left(\frac{\mu_A - E_0}{E_1} \right) + V_0 \\ &= \mu_A^2 \left(\frac{V_2}{E_1^2} \right) + \mu_A \left(-\frac{2E_0 V_2}{E_1^2} + \frac{V_1}{E_1} \right) + \left(\frac{E_0^2 V_2}{E_1^2} - \frac{E_0 V_1}{E_1} + V_0 \right) \\ &= A_2 \mu_A^2 + A_1 \mu_A + A_0 \\ &= A_2 \left(\mu_A + \frac{A_1}{2A_2} \right)^2 - \frac{A_1^2}{4A_2} + A_0 \\ &= 1599.714 (\mu_A - 0.06913354)^2 + 1.751926\end{aligned}$$

where

$$\begin{aligned}A_2 &= \frac{V_2}{E_1^2} = 1599.714 \\ A_1 &= -\frac{2E_0 V_2}{E_1^2} + \frac{V_1}{E_1} = -221.1878 \\ A_0 &= \frac{E_0^2 V_2}{E_1^2} - \frac{E_0 V_1}{E_1} + V_0 = 9.397673.\end{aligned}$$

Therefore, the new graph is



Note that the new curve will only touch the efficient frontier at the point of market portfolio since it can be verified that the optimal portfolio of mean return μ_D is $(0.152, 0.495, 0.353)^T \neq (0.25, 0.25, 0.5)^T$. Thus D is not an optimal portfolio and the any combinations of D and M are not optimal except when we allocate all the weights on the market portfolio.

Alternatively, one may observe that in that case where we consider two assets only, any linear combinations of the two assets must be optimal. Because it is impossible to have two portfolio with different weightings but the same mean return. Hence, the theories from Chapter 1 can be directly applied to D and M.

Therefore, the volatility of the linear combinations can be found using Theorem 2 from Chapter. But you would still need to substitute out μ_A from the Theorem to get an expression of σ_A in terms of a_D .

Two marks are given for mean and variance of asset D. Three marks for the correct curve. Three marks for derivation of mean and variance of the combinations of D and tangency portfolio (correct placement of D: 1 mark, the curve touches tangency portfolio: 1 mark, the curve is below EF from (b): 1 mark).

3. a) Denote c_i as the market capitalization of asset i . $SMB = \mathcal{R}_{S,t} - \mathcal{R}_{B,t}$, where $\tilde{c} = \text{median}_{i=1, \dots, n}(c_i)$, $\mathcal{R}_{S,t} = \sum_{i=1}^n c_i \mathbb{1}(c_i \leq \tilde{c}) R_{i,t} / (\sum_{i=1}^n c_i \mathbb{1}(c_i \leq \tilde{c}))$ and $\mathcal{R}_{B,t} = \sum_{i=1}^n c_i \mathbb{1}(c_i > \tilde{c}) R_{i,t} / (\sum_{i=1}^n c_i \mathbb{1}(c_i > \tilde{c}))$. Denote z_i as the book-to-market ratio of asset i . $HML = \mathcal{R}_{H,t} - \mathcal{R}_{L,t}$, where q_α is the α th quantile of the book-to-market ratios $(z_1, \dots, z_n)$ of all the assets, $\mathcal{R}_{H,t} = \sum_{i=1}^n c_i \mathbb{1}(z_i > q_{0.7}) R_{i,t} / (\sum_{i=1}^n c_i \mathbb{1}(z_i > q_{0.7}))$ and $\mathcal{R}_{L,t} = \sum_{i=1}^n c_i \mathbb{1}(z_i \leq q_{0.3}) R_{i,t} / (\sum_{i=1}^n c_i \mathbb{1}(z_i \leq q_{0.3}))$.

for each factor: 1 mark for the name of performance index. 1 mark for thresholds. 1 mark for formula.

-1 mark atmost if the notations are not clearly defined

b)

$$\hat{R}_{At} - r_t = 0.001 + 0.981\text{ReM} + 0.422\text{SMB} + 1.935\text{HML}$$

The factor loadings are $\hat{\mathbf{b}}_A = (b_{A,1}, b_{A,2}, b_{A,3}) = (0.981, 0.422, 1.935)$.

- c) Dentoe that vector containing factor loadings for a stock I as $\hat{\mathbf{b}}_I$. Suppose the returns of all stocks at a fixed time t is stored in

$$\mathbf{Y}_t = \begin{pmatrix} R_{A,t} \\ R_{B,t} \\ R_{C,t} \end{pmatrix},$$

and $\mathbf{X}_t$ takes the following form

$$\mathbf{X}_t = \begin{pmatrix} 1 & \hat{\mathbf{b}}_A^T \\ 1 & \hat{\mathbf{b}}_B^T \\ 1 & \hat{\mathbf{b}}_C^T \end{pmatrix}$$

Then for time t , one can estimate the $\lambda_t = (\lambda_{0,t}, \dots, \lambda_{3,t})^T$ by regressing Y_t by X_t , i.e. $\hat{\lambda}_t = (X_t^T X_t)^{-1} X_t^T Y_t$.

The final estimates are $\bar{\lambda} = \sum_{t=1}^T \hat{\lambda}_t / T$

Credits will be given if students are able to clearly describe the regression procedure, i.e.

- regression is done for each time point t .
 - state the dependent variable and explanatory variable.
 - any indication that there is an intercept term is needed.
 - notations are defined clearly.
- d) The above regression is not feasible, since for each time point t , we only have three observations that can be used to estimate λ_t , which is too few.
- partial credits may be given for answers that are not completely wrong.

2 Part II: Theory

1. a) By Theorem 1 in Chapter 1

$$\begin{aligned}\tilde{c}_{11} &= \mathbf{1}^T \tilde{\Sigma}^{-1} \mathbf{1} \\ \tilde{c}_{1\mu} &= \mathbf{1}^T \tilde{\Sigma}^{-1} \tilde{\boldsymbol{\mu}} \\ \tilde{c}_{\mu\mu} &= \tilde{\boldsymbol{\mu}}^T \tilde{\Sigma}^{-1} \tilde{\boldsymbol{\mu}} \\ \tilde{\Delta} &= \tilde{c}_{11} \tilde{c}_{\mu\mu} - \tilde{c}_{1\mu}^2\end{aligned}$$

- b) Denote $\mathbf{b} = (b_1, \dots, b_n)^T$

$$\begin{aligned}v\mathbf{b}^T &= v(b_1, \dots, b_n) \\ &= (\sigma_{01}, \dots, \sigma_{0n}) \\ &= (\rho_1 v \sqrt{\sigma_{11}}, \dots, \rho_n v \sqrt{\sigma_{nn}})\end{aligned}$$

Therefore

$$\begin{aligned}\mathbf{b}^T &= (b_1, \dots, b_n) = (\rho_1 \sqrt{\sigma_{11}}, \dots, \rho_n \sqrt{\sigma_{nn}}). \\ \Sigma &= (\sigma_{ij})_{1 \leq i, j \leq n}.\end{aligned}$$

1 mark for each formula.

- c) Denote $\mathbf{I}$ as an identity matrix with its dimension implicitly known and $\mathbf{0}$ as a vector of 0 with its dimension implicitly known.

$$\begin{aligned}& \begin{pmatrix} av^{-2} & -av^{-1}\mathbf{b}^T \Sigma^{-1} \\ -av^{-1}\Sigma^{-1}\mathbf{b} & \Sigma^{-1} + a\Sigma^{-1}\mathbf{b}\mathbf{b}^T \Sigma^{-1} \end{pmatrix} \tilde{\Sigma} \\ &= \begin{pmatrix} av^{-2} & -av^{-1}\mathbf{b}^T \Sigma^{-1} \\ -av^{-1}\Sigma^{-1}\mathbf{b} & \Sigma^{-1} + a\Sigma^{-1}\mathbf{b}\mathbf{b}^T \Sigma^{-1} \end{pmatrix} \begin{pmatrix} v^2 & v\mathbf{b}^T \\ v\mathbf{b} & \Sigma \end{pmatrix} \\ &= \begin{pmatrix} a - a\mathbf{b}^T \Sigma^{-1} \mathbf{b} & av^{-1}\mathbf{b}^T - av^{-1}\mathbf{b}^T \\ -av\Sigma^{-1}\mathbf{b} + v\Sigma^{-1}\mathbf{b} + av\Sigma^{-1}\mathbf{b}\mathbf{b}^T \Sigma^{-1}\mathbf{b} & -a\Sigma^{-1}\mathbf{b}\mathbf{b}^T + \mathbf{I} + a\Sigma^{-1}\mathbf{b}\mathbf{b}^T \mathbf{I} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ -av\Sigma^{-1}\mathbf{b}(1 - \mathbf{b}^T \Sigma^{-1} \mathbf{b}) + v\Sigma^{-1}\mathbf{b} & \mathbf{I} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ -v\Sigma^{-1}\mathbf{b} + v\Sigma^{-1}\mathbf{b} & \mathbf{I} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \\ &= \mathbf{I}\end{aligned}$$

1 mark for each entries.

- d) Note that from c), we know that

$$\tilde{\Sigma}^{-1} = \begin{pmatrix} av^{-2} & -av^{-1}\mathbf{b}^T \Sigma^{-1} \\ -av^{-1}\Sigma^{-1}\mathbf{b} & \Sigma^{-1} + a\Sigma^{-1}\mathbf{b}\mathbf{b}^T \Sigma^{-1} \end{pmatrix}$$

The following answers make use of the following facts: Suppose

$$A = \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{pmatrix} \quad B = \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{pmatrix} \quad M = \begin{pmatrix} D_1 & D_2 \\ D_3 & D_4 \end{pmatrix},$$

where $\mathbf{a}_i$ and $\mathbf{b}_i$ are vectors of length ℓ_{ai} and ℓ_{bi} such that A and B are of length n , D_1 is $\ell_{a_1} \times \ell_{b_1}$, D_2 is $\ell_{a_1} \times \ell_{b_2}$, D_3 is $\ell_{a_2} \times \ell_{b_1}$ and D_4 is $\ell_{a_2} \times \ell_{b_2}$ such that M is $n \times n$.

$$\begin{aligned} A^T M B &= (\mathbf{a}_1^T \quad \mathbf{a}_2^T) \begin{pmatrix} D_1 & D_2 \\ D_3 & D_4 \end{pmatrix} \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{pmatrix} \\ &= (\mathbf{a}_1^T \quad \mathbf{a}_2^T) \begin{pmatrix} D_1 \mathbf{b}_1 + D_2 \mathbf{b}_2 \\ D_3 \mathbf{b}_1 + D_4 \mathbf{b}_2 \end{pmatrix} \\ &= \mathbf{a}_1^T D_1 \mathbf{b}_1 + \mathbf{a}_1^T D_2 \mathbf{b}_2 + \mathbf{a}_2^T D_3 \mathbf{b}_1 + \mathbf{a}_2^T D_4 \mathbf{b}_2 \end{aligned}$$

Students should be aware that the order of the multiplications for matrix and vector operations is important.

Note also if M is symmetric, then $A^T M B = (A^T M B)^T$ since it is a scalar. Since $(A^T M B)^T = B^T (A^T M)^T = B^T M A$. Thus $c_{1b} = c_{b1}$, $c_{1\mu} = c_{\mu 1}$ and $c_{\mu b} = c_{b\mu}$.

Direct applications of the above results should yield the answers.

$$\begin{aligned} \tilde{c}_{11} &= \mathbf{1}^T \tilde{\Sigma}^{-1} \mathbf{1} \\ &= av^{-2} - av^{-1}c_{1b} - av^{-1}c_{1b} + c_{11} + ac_{1b}^2 \\ &= av^{-2}(1 - 2vc_{1b} + v^2c_{1b}^2) + c_{11} \\ &= av^{-2}(1 - vc_{1b})^2 + c_{11} \\ \tilde{c}_{1\mu} &= \mathbf{1}^T \tilde{\Sigma}^{-1} \hat{\boldsymbol{\mu}} \\ &= av^{-2}r - av^{-1}c_{b\mu} - av^{-1}rc_{1b} + c_{1\mu} + ac_{1b}c_{b\mu} \\ &= av^{-2}(r - vc_{b\mu} - vrc_{1b} + v^2c_{1b}c_{b\mu}) + c_{1\mu} \\ &= av^{-2}(r - vc_{b\mu})(1 - vc_{1b}) + c_{1\mu} \\ \tilde{c}_{\mu\mu} &= \hat{\boldsymbol{\mu}}^T \tilde{\Sigma}^{-1} \hat{\boldsymbol{\mu}} \\ &= av^{-2}r^2 - av^{-1}rc_{b\mu} - av^{-1}rc_{b\mu} + c_{\mu\mu} + ac_{b\mu}^2 \\ &= av^{-2}(r^2 - 2vrc_{b\mu} + v^2c_{b\mu}^2) + c_{\mu\mu} \\ &= av^{-2}(r - vc_{b\mu})^2 + c_{\mu\mu} \end{aligned}$$

1 mark will be deducted if any non scalar terms are found in intermediate steps.

- e) Note that $g(x) = O(f(x))$ means there exist a positive number M and a real x_0 such that $\frac{g(x)}{f(x)} \leq M$ for $x > x_0$.

$$\begin{aligned} \tilde{c}_{11} &= av^{-2}(1 - 2vc_{1b} + v^2c_{1b}^2 + a^{-1}v^2c_{11}) \\ &= av^{-2} + O(v^{-1}) \\ \tilde{c}_{1\mu} &= av^{-2}(r - vc_{b\mu} - vrc_{1b} + v^2c_{1b}c_{b\mu} + a^{-1}v^2c_{1\mu}) \\ &= av^{-2}r + O(v^{-1}) \\ \tilde{c}_{\mu\mu} &= av^{-2}(r^2 - 2vrc_{b\mu} + v^2c_{b\mu}^2 + a^{-1}v^2c_{\mu\mu}) \\ &= av^{-2}r^2 + O(v^{-1}) \\ \tilde{\Delta} &= \left\{ av^{-2}(1 - vc_{1b})^2 + c_{11} \right\} \left\{ av^{-2}(r - vc_{b\mu})^2 + c_{\mu\mu} \right\} - \left\{ av^{-2}(r - vc_{b\mu})(1 - vc_{1b}) + c_{1\mu} \right\}^2 \\ &= av^{-2} \left\{ c_{11}(r - vc_{b\mu})^2 + c_{\mu\mu}(1 - vc_{1b})^2 - 2c_{1\mu}(r - vc_{b\mu})(1 - vc_{1b}) + a^{-1}v^2c_{11}c_{\mu\mu} + a^{-1}v^2c_{1\mu}^2 \right\} \\ &= av^{-2} \left\{ c_{11}r^2 + c_{\mu\mu} - 2c_{1\mu}r \right\} + O(v^{-1}) \end{aligned}$$

- f) Substituting all expression from d) and e) back to (1). One should obtain

$$\begin{aligned} \tilde{\Delta}^{-1}(\tilde{c}_{11}\tilde{\Sigma}^{-1}\tilde{\boldsymbol{\mu}} - \tilde{c}_{1\mu}\tilde{\Sigma}^{-1}\mathbf{1})\mu_* &= \left(\frac{\mu_*(rc_{11} - c_{1\mu})}{c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}} + O(v) \right) \\ &\quad \left(\frac{\mu_*(\Sigma^{-1}\boldsymbol{\mu} - r\Sigma^{-1}\mathbf{1})}{c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}} + O(v) \right) \\ \tilde{\Delta}^{-1}(\tilde{c}_{\mu\mu}\tilde{\Sigma}^{-1}\mathbf{1} - \tilde{c}_{1\mu}\tilde{\Sigma}^{-1}\tilde{\boldsymbol{\mu}}) &= \left(\frac{c_{\mu\mu} - rc_{1\mu}}{c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}} + O(v) \right) \\ &\quad \left(\frac{r^2\Sigma^{-1}\mathbf{1} - r\Sigma^{-1}\boldsymbol{\mu}}{c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}} + O(v) \right) \end{aligned}$$

Therefore

$$\mathbf{w}_*(\mu_*) = \left(\frac{\frac{c_{\mu\mu} - c_{1\mu}(r + \mu_*) + r\mu_*c_{11}}{c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}}}{\frac{(\mu_* - r)(\Sigma^{-1}\boldsymbol{\mu} - r\Sigma^{-1}\mathbf{1})}{c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}}} + O(v) \right) \xrightarrow{v \rightarrow 0} \left(\frac{\frac{c_{\mu\mu} - c_{1\mu}(r + \mu_*) + r\mu_*c_{11}}{c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}}}{\frac{(\mu_* - r)(\Sigma^{-1}\boldsymbol{\mu} - r\Sigma^{-1}\mathbf{1})}{c_{\mu\mu} - 2rc_{1\mu} + r^2c_{11}}} \right)$$

We can see that it reduces to the solution of optimal portfolio under the existence of risk-free asset which is natural since $v \rightarrow 0$ means asset 0 becomes risk-free asymptotically.

Partial credits will be given as long as students are able to point out that the portfolio will converge to the optimal portfolio with risk-free assets.

2 marks are given at most for incorrect attempts to solve the problem.