RMSC 4003 Statistical Modelling in Financial Markets

Midterm

9:30-12:15. Tuesday, 20 Oct 2026

Name: _____ **Major:** _____

Part I: Applications: (70 marks):

1) (12 marks) A researcher is conducting Fama-French analysis. He stored a vector of historical returns of a portfolio in the `R`-object `ReP`. Also, he obtained the historical returns of the market index `ReM`, and three variables `SMB`, `HML`, and `CMA`. The risk-free rate is `r`.

   a) (1 marks) How can you use `R` to find the sample size of this data set?

   b) (6 marks) State the full names of the variables `SMB`, `HML`, and `CMA`, and state the key financial quantities that each variable refers to.

   c) (3 marks) Suppose that the operating profitability and market capitalization of all stocks in the market are stored in the vectors `Z` and `W`, respectively. Also, the historical returns of all stocks in the markets are stored in the matrix `Re`, where each column represents the historical returns of one stock. Write down the `R`-codes to compute the variable `RMW`.

   d) (2 marks) Write down the `R`-command to fit the Fama-French model for `ReP`.

2) (58 marks) Let the risk free rate be $r = 0.008$. Suppose that the estimated expected returns and covariance matrix of returns for three stocks A, B, C, and the inverse of the covariance matrix are

$$\hat{\boldsymbol{\mu}} = \begin{pmatrix} 0.02 \\ 0.01 \\ 0.03 \end{pmatrix}, \quad \hat{\boldsymbol{\Sigma}} = \begin{pmatrix} 0.2 & 0.1 & -0.1 \\ 0.1 & 0.3 & 0.1 \\ -0.1 & 0.1 & 0.25 \end{pmatrix}, \quad \hat{\boldsymbol{\Sigma}}^{-1} = \begin{pmatrix} 11.8 & -6.36 & 7.27 \\ -6.36 & 7.27 & -5.46 \\ 7.27 & -5.46 & 9.09 \end{pmatrix}.$$

   It is given that $\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{1} = (12.7, -4.54, 10.91)'$ and $\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\mu}} = (0.39, -0.22, 0.36)'$.

   a) (10 marks) Find the optimal portfolio (using only risky assets) that achieves an expected return of 0.03. What is the variance of this portfolio?

   b) (10 marks) Draw the efficient frontier constructed from the three stocks. Indicate Stocks A to C on the graph.

   c) (4 marks) Find the overall minimum risk portfolio using only risky assets.

   d) (5 marks) Give the `R`-commands that find the optimal portfolio achieving an expected return of 0.025, under the condition that no short-selling is allowed.

   e) (5 marks) Find the tangency portfolio $\boldsymbol{w}_M$, and give its expected return and volatility.

   f) (4 marks) Fill in the blanks: One Fund Theorem states that any portfolio on the 1)_____ can be constructed from a 2)_____ combination of 3)_____ and 4)_____.

   g) (5 marks) Construct the optimal portfolio with target return 0.06 using 3) and 4) in the above question.

h) (3 marks) Assume that $\boldsymbol{w}_M$ in e) is the market portfolio, find the CAPM β of Stock A.

i) (4 marks) Find the Sharpe ratio and Treynor ratio of Stock A.

j) (5 marks) Explain why the tangency portfolio is also called the market portfolio. Is the assumption in h) appropriate? Why?

k) (3 marks) Suppose that the initial price of Stock A and the market portfolio are \$ 2.2 and \$ 120, respectively. Discuss how to hedge the risk of holding 1000 units of Stock A.

Part II: Theory: (30 marks)

3) (26 marks) Consider three assets A, B, C in the market. Denote their returns by the vector $\boldsymbol{R} = (R_1, R_2, R_3)'$, with $\boldsymbol{\mu} = E(\boldsymbol{R}) = (2,3,4)'$ and

$$\Sigma = \mathrm{Var}(\boldsymbol{R}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix},$$

Consider the optimization problem: for a fixed target return level μ^*, we find the optimal portfolio of weights

$$\boldsymbol{w}^* = (w_1^*, w_2^*, w_3^*)' = \arg\min_{\boldsymbol{w}} \boldsymbol{w}'\Sigma\boldsymbol{w} \quad \text{subject to } \boldsymbol{w}'\boldsymbol{\mu} = \mu^* \text{ and } \boldsymbol{w}'\mathbf{1} = 1, \tag{1}$$

for some pre-specified μ^*.

a) (6 marks) Find the optimal portfolio $\boldsymbol{w}^*$ that gives a target return of $\mu^* = 4$. What is the volatility of this portfolio?

b) (4 marks) Show that, for any μ^*, the optimal portfolio has the same volatility as the one in a).

c) (2 marks) Draw the efficient frontier (only risky assets) of the market.

d) (2 marks) Pascal claims that "Stock B and C have the same covariance structure but Stock C has a higher expected return. So one should abandon B and invest in Stock C only. Thus, $w_2^* = 0$". Does his claim agree with the answer in a)? Explain.

e) (2 marks) Jerry claims that "Stocks B and C form an arbitrage pair, one can enjoy an arbitrary high return. Thus, for a fixed return level, the optimal risk can be 0. Does his claim agree with the answer in a)? Explain.

f) (2 marks) Parco claims that "Even if the expected return of Stock C is not 4, the volatility obtained in a) is still the same.". Do you agree with him? Explain.

g) (6 marks) If the risk free assest with $r = 0.5$ is allowed to be used to construct the optimal portfolio, find the optimal portfolio $\boldsymbol{w}^* = (w_1^*, w_2^*, w_3^*, w_r^*)'$ that gives a target return of μ^*. Here w_r^* is the weight assigned to the risk free asset.

h) (2 marks) Draw the efficient frontier (risky and risk-free assets) of the market .

4) (4 marks) Can the CAPM beta of a portfolio be negative? Is CAPM beta bounded above by a constant? State the answer and briefly explain.

End of paper